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A SIMPLIFIED STUDY OF OPEN-CYCLE MAGNETOHYDRODYNAMIC POWER GENERATION AND AN EXAMINATION OF THE POSSIBLE ROLE OF SUPERSONIC COMBUSTION IN THE CYCLE

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(9) Technical rept.,

SUMMARY

This Report presents a preliminary simplified study of some of the facets involved in obtaining open-cycle M.H.D. power generation. Various parts of an idealised cycle are studied in detail, and examination has been made of the possibility of supersonic combustion forming a natural feature of M.H.D. power generation cycles in which a supersonic M.H.D. generating duct is used.

In Appendices A to E, for completeness, formulae relevant to the various stages of the cycle are listed or derived.

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Departmental Reference: Aero ~~2892~~

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CONTENTS

1 INTRODUCTION - IDEALISED M.H.D. POWER GENERATION CYCLE

Fig.1(a) shows an idealised M.H.D. power generation cycle, in which the compressor, nozzle and diffuser of the system are assumed to operate isentropically. In this cycle it is also assumed that no recompression is necessary after the diffuser (following heat regeneration) in order to exhaust to the atmosphere or alternatively pass the gases through a conventional steam boiler before rejection. This figure illustrates how entropy and ambient enthalpy (temperature) vary through the cycle. In general the gas-velocity will be low for the sections 0-1, 1-2, and 6-7-0. For the combustion process 2-3 no restriction to subsonic flow has been assumed, and, in fact, the only restriction for this stage is that in practice the ambient temperature (enthalpy) may well be limited (see section 3).

Fig.1(b), on the other hand, shows the same cycle as Fig.1(a) but here the total enthalpy and total pressure required in the system are shown. In this diagram the isentropic sections assumed above are represented by a single point in each case, and this representation of the cycle shows clearly how the total pressure in the system is reduced progressively, following the initial compression in the leg 0-1, back to atmospheric pressure at 0. Also the sections of the cycle are clearly seen where the total enthalpy, and hence the driving enthalpy for convective heat transfer, is high.

1.1 M.H.D. duct temperature and heat regeneration

In order to get the degree of conductivity in the gas, even with seeding[1], which would be sufficient for economic use in the M.H.D. duct, the ambient temperature in the duct would have to be of the order of 2500°K[1,2,3,4,5]. Although with stoichiometric mixtures of fossil fuel and air at atmospheric temperature it would appear to be possible to obtain temperatures of this order by combustion, the adiabatic temperature of fossil fuel being around 2300-2700°K, in practice such temperatures are only obtained by having oxygen enrichment of the air or using air preheated (by regeneration) by about 1000°C. Hence when oxygen enrichment proves uneconomic heat regeneration forms an essential part of the cycle.

1.2 Thring's idea of improved heat regeneration leading to 'hypersonic' combustion

The maximum temperature at which a metallic wall capable of standing a very high pressure difference can be operated at the present time is about 1000 to 1100°C. It is therefore possible to regenerate heat through a wall

maintained at this temperature. In view of this Thring[3,5] has proposed a heat exchanger which "would not only pre-heat the air to 1000°C, but then would be extended to a series of convergent/divergent nozzles in parallel which were externally heated by the combustion products, so that the heat could be passed through the walls and the expansion to high velocity could take place essentially isothermally at 1000°C. In this way one arrives at a gas having the necessary velocity and at 1000°C, but not yet with the fuel burnt in it". Thring goes on to point out that by this method one has transferred considerably more heat into the system at 1000°C, due to the additional intake of kinetic energy, than would be possible, say, in the ordinary system where the air velocity at the entry to the combustion chamber is low. Thus he submits that combustion can then take place in the high velocity, possibly hypersonic, stream. Indeed this system would give a net boost to the total enthalpy of the working gas in the combustion chamber, with the only penalty, a more severe loss of total pressure through supersonic/hypersonic combustion compared to subsonic combustion (see section 4.1), which would have to be allowed for in the initial compression stage of the cycle. However, it is not clear how this convergent/divergent - honeycomb heat exchanger is to work. For, although the wall temperature may be maintained at 1000°C and the cooler stream expanded rapidly, heat transfer from the wall to the gas will be dominated by the total enthalpy of the cooler stream at each station rather than its ambient enthalpy (temperature). Hence the effectiveness of heat-exchangers of this type might be strictly limited in practice.

2 OPEN-CYCLE OPERATION WITHOUT HEAT REGENERATION

Although heat regeneration may be an essential part of the M.H.D. power generation cycle (see section 1.1), in order to study in more detail the interdependence of the combustion, nozzle, and M.H.D. duct parts of the cycle, it is convenient to include heat regeneration with combustion heat as in Fig.2 rather than to treat it as a separate stage of the cycle as in Fig.1. This can be justified if one considers either the case of combustion with oxygen enrichment of the air and no heat regeneration, or more generally that for this simplified model of the processes the heat added by regeneration and the heat released by combustion may be included under the heading of "combustion" alone. Hence for this latter case if we now define the heat released by combustion, q, by:

$$q = q_R + q_C \tag{1}$$

(c.f. Figs.1 and 2)

then, since the thermal efficiency, η_{th}, of the cycle in Fig.1 with heat regeneration is given by:

$$\underset{\text{with regen.}}{\eta_{th}} = \frac{P - w}{q_C} \tag{2}$$

the thermal efficiency of the cycle in Fig.2 considered from the view of no heat regeneration will be related by:

$$\underset{\text{with regen.}}{\eta_{th}} = \underset{\text{without regen.}}{\eta_{th}} \left(\frac{q}{q_C}\right)$$

$$= \underset{\text{without regen.}}{\eta_{th}} \left(\frac{q_R + q_C}{q_C}\right) . \tag{3}$$

The duct efficiency will of course be unaltered by this device and is given for both cases by:

$$\eta_{duct} = \frac{P}{(h_{t_o} + w + q)} = \frac{P}{(h_{t_o} + w + q_R + q_C)} \left(= \frac{\text{Energy from duct}}{\text{Total energy at entry to duct}}\right) . \quad \ldots \tag{4}$$

Thus by this method we see that we may consider the problem neglecting heat regeneration, but we must consider in its place more efficient combustion than is usually possible, and remember that the thermal efficiency of the cycle with heat regeneration will in practice be much improved upon that without regeneration.

It should be noted that in order to make the assumptions above, the working fluid in the cycle has been idealised, and has been considered to be composed of combustion products throughout all stages of the cycle. Hence a constant value for γ, the ratio of specific heats, for the working gases has been assumed and the fuel mass addition has been neglected. The combustion process (and heat generation) thus takes the form of a latent heat release in the system, or may be considered as heat energy transferred to the gases during combustion.

2.1 Typical relationships between the cycle efficiencies

From equations (1), (2), (3) and (4) we see that for the cycle given in Fig. 2 we have:

$$\eta_{\substack{th \\ \text{without regen.}}} = \frac{P - w}{q} = \left(\frac{h_{t_o} + w + q}{q}\right)\eta_{duct} - \left(\frac{w}{q}\right) \quad . \quad (5)$$

For a typical range of conditions generally quoted for this cycle, it is shown in Appendix A that, as the initial compression ratio, C, between states 0 and 1 increases from 1 to 16, equation (5) varies in the form:

$$\eta_{\substack{th \\ \text{without regen.}}} = \begin{pmatrix} 1.125 \\ \text{to} \\ 1.285 \end{pmatrix} \eta_{duct} - \begin{pmatrix} 0 \\ \text{to} \\ 0.143 \end{pmatrix} \quad .$$

Hence for reasonable duct efficiencies the thermal efficiency in Fig. 2 will for these conditions be close to the duct efficiency, assuming of course that the compression ratio in turn is not excessive. However, high initial compression ratios may be required if high pressure losses occur in the cycle. (see section 4), and the above relationship will no longer apply (see Appendix E and Fig. 7).

It is further shown in Appendix A that the thermal efficiency with regeneration will be around 1.6 to 1.4 times higher than the above value for the same range of the initial compression ratio.

3 EFFECTS OF AMBIENT TEMPERATURE LIMITATIONS IN THE COMBUSTION (REGENERATION) - NOZZLE LEG OF THE CYCLE

As indicated in section 1.1 the total enthalpy of the system after combustion may be limited in practice, due to heat losses in the combustion chamber. In order to simplify the picture for the energy-exchange processes in the combustion chamber, these losses may be replaced by considering instead inefficient or incomplete combustion. However, even with practical limitations on the total enthalpy which the system may attain, it may still be necessary to restrict the ambient enthalpy (temperature) throughout the cycle, and in particular in the combustion chamber. This in general will not be the result of restricting convective heat transfer, since, as mentioned earlier, this

depends more directly on the total enthalpy of the system. Nor is it expected to be necessary to restrict ambient enthalpy on account of radiation losses, although radiative energy transfer increases rapidly with increase of ambient temperature, since the ambient temperature ranges considered at present for practical purposes are still below the threshold for significant radiative effects to occur in the gas stream. But a restriction on ambient temperature may result from attempting to limit or control the chemistry in the combustion reactions. Typically this might come from the necessity to avoid dissociations of the working gas (ionization coming from seeding) or influence the relative concentrations of combustion products. In this latter category, for example, we have the suggestion for incomplete combustion of fuel[4] in order to reduce the amount of CO_2 as compared to CO, since at high temperatures CO_2 would have more serious erosive chemical effect on the electrodes of the duct.

3.1 Supersonic combustion due to ambient enthalpy limitations

In a gas stream the ratio of total enthalpy to ambient enthalpy is given approximately by:

$$\frac{h_t}{h} = 1 + \frac{(\gamma - 1)}{2} M^2 \quad .$$

Hence the gas stream is supersonic when:

$$\frac{h_t}{h} > \frac{(\gamma + 1)}{2} \quad . \tag{6}$$

For values of γ in the range 1.2 to 1.3 this latter formula shows that a gas stream will be supersonic when the total enthalpy of the stream is more than 1.1 to 1.15 times greater than the ambient enthalpy. Therefore, if a supersonic M.H.D. duct is to be used in the generation cycle and the ambient enthalpy level is restricted during combustion, these values for enthalpy ratio may well be exceeded at some point in the combustion chamber. Hence the combustion process may well be required to consist of part subsonic and part supersonic combustion.

In Fig. 2, since suffix 4 denotes entry conditions to the M.H.D. generating duct, the inequality (6) above shows that supersonic combustion will occur in the combustion chamber whenever the maximum ambient enthalpy, h_{max}, satisfies the inequality:

$$h_{max} < h_4 \left(\frac{2 + (\gamma - 1) M_4^2}{\gamma + 1}\right) \quad . \tag{7}$$

As mentioned earlier in section 1.1, h_4 should correspond to at least a temperature T_4 of around 2500°K. Hence for $\gamma = 1.2$, (7) indicates that for supersonic flow during combustion T_{max} would have to be less than 2500°K (i.e. h_4) for $M_4 = 1$ and 3180°K (i.e. 14/11 h_4) for $M_4 = 2$. The corresponding values for $\gamma = 1.3$ are 2500°K for $M_4 = 1$ and 3480°K for $M_4 = 2$. Although these calculations assume constant specific heats and thermally perfect gases, the rough values quoted serve to illustrate how, for a supersonic duct, supersonic combustion may still be necessary at some stage, even when the ambient temperature in the combustion chamber is allowed to be as high as 3000°K.

4 PRESSURE LOSSES IN THE COMBUSTION (REGENERATION) - NOZZLE LEG OF THE CYCLE

Using the formula derived in Appendix B, we find quite generally that the change (loss) in total pressure developed from state 1 to state 4 in the cycle illustrated in Fig. 2 is given from (16) by:

$$\left(\frac{p_{t_4}}{p_{t_1}}\right)^{\frac{\gamma-1}{\gamma}} = \left(1 + \frac{q}{h_{t_1}}\right) e^{-\int_0^q \frac{dq}{h}} < \left(1 + \frac{q}{h_{t_1}}\right) e^{-\frac{q}{h_{max}}} \quad .$$

Hence for a given amount of heat addition to the flow we see that the loss in total pressure has a lower limit which depends on the maximum ambient enthalpy (temperature) that can be allowed for the combustion process (i.e. the process which develops the minimum gain in entropy).

Optimisation of the power generation cycle therefore requires the efficiency of the combustion process to be as close as possible to this lower limit in order to minimise this loss in total pressure, which, as is seen in section 2, has to be compensated for in the initial compression stage 0-1 of the cycle.

4.1 Model of the combustion (regeneration) - nozzle processes

Fig. 3 shows a model of the processes between states 1 (or 2) and 4 in Figs. 1 and 2, where intermediate states A, B, C and D have been added. Expansion of the flow isentropically prior to combustion (or heat addition) as well as expansion before entry to the M.H.D. duct have been included in this model.

Fig.3(a) shows that once the maximum allowable ambient enthalpy has been reached at B the combustion process could continue in general for this model to C at the same enthalpy, reducing in ambient enthalpy then to D' before isentropically expanding to 4'. However, since this process will by the formula above be less efficient than continuing the path from C to D at maximum ambient enthalpy before isentropic expansion to 4, the paths to the dashed points will not be considered, and the point C may be ignored in the model of the cycle shown.

Appendix C gives the relationships required to determine the various states through the isentropic nozzle processes, 1 (or 2) - A and D - 4, and the combustion processes A-B and B-D. In the latter category we have constant ambient enthalpy combustion for B-D. The combustion process A-B is, however, not defined and for this one may assume, as in Appendix C, constant pressure (velocity), constant Mach number, constant area (which would include combustion through a detonation wave), or some more general combustion process, determining properties in this latter case by using the basic formulae of Appendix B.

Comparative study of the loss of total pressure required by the combustion-plus-nozzles model illustrated in Fig.3 can thus be made by defining h_{max} (which in some cases may be taken to be so large that in fact no ambient temperature restriction exists) and varying both the combustion process A-B and the velocity at state A. The velocity at state 4, the conditions at state 1 (or 2), and the total combustion heat release being fixed for this investigation, (the parameter λ being fixed automatically by the other assumptions). Again from the formula for the overall loss of total pressure given earlier in this section, it is obvious that the minimum loss of total pressure (minimum gain in entropy) will result when the combustion path A-B is chosen such that the ambient enthalpy (temperature) at each stage of the process is as large as possible (i.e. as close to h_{max} as possible).

4.2 Simplified calculations based on the above model allowing for completely supersonic and completely subsonic combustion

Fig.4 shows the results from calculations (made by Thompson) based on the model in Fig.3 for a fixed amount of heat addition by combustion where the combustion-nozzle processes between states 2 and 4 have been considered. (i.e. only the processes upstream of the M.H.D. duct which occur after the heat regeneration leg 1-2 have been considered.) Also no restriction on

ambient enthalpy level in the combustion process has been made. Both completely supersonic and completely subsonic combustion processes have been considered together with the appropriate isentropic nozzle expansions which are required to match in with these in order to arrive at the same Mach number, $M_4 = 1.4$, at entry to the M.H.D. duct.

The key to the various combustion-nozzle combinations studied is given in Fig.4(a), with the various paths $2\text{-}I_{(n)}\text{-}4$ identified by the suffix, (n), of the intermediate state $I_{(n)}$. For the conditions of this example it was not possible to include the path with isentropic expansion followed by supersonic combustion at constant area. The reason for this is that in this case the initial total enthalpy at state 2 was not sufficiently high to allow expansion of the flow to the large velocity required for matching this process to constant area supersonic combustion through to a Mach number of 1.4 at state 4.

Fig.4(b) illustrates clearly for this example the large loss of total pressure in the system through such supersonic combustion processes. As mentioned earlier, such pressure losses would thus have to be tolerated in the Thring model of the heat regeneration - combustion system described in section 1.2.

Fig.5 shows results from calculations similar to those illustrated in Fig.4, where the Mach number at entry to the duct was assumed to be 2 instead of 1.4. Again matching supersonic combustion at constant area to the required isentropic nozzle expansion was not possible for the conditions assumed. Nor was this matching in fact possible for supersonic combustion at constant pressure, although apart from this the general picture is much the same as before in Fig.4.

The processes shown in Figs.4 and 5 are by no means optimum, but they illustrate the large pressure losses involved with having fully supersonic combustion in the cycle as compared to fully subsonic combustion.

5 CONSTANT MACH NUMBER M.H.D. DUCT - INTEGRATION OF THE DUCT INTO THE CYCLE

In order to estimate the efficiency of the M.H.D. duct itself, the duct with constant Mach number has been considered. In fact, as is shown in Refs.6 and 7, this duct is close to having optimum efficiency, even when a constant load factor, K (see Appendix D), is assumed.

A summary of the relations appropriate to this duct are given in Appendix D, and the efficiency, η_{duct}, for such a system is illustrated in Fig.6. In this figure two duct Mach numbers have been chosen together with two values of the load factor, K. It should be noted how sharply (almost exponentially) the pressure ratio required across the duct rises with a demand for increase in efficiency. In order to integrate the duct into a complete cycle it is therefore obvious that a large total pressure loss will have to be allowed for in the duct itself, and this means that loss of total pressure will thus be at a high premium throughout the early part of the cycle upstream of the duct.

5.1 Integration of the constant Mach number duct into the complete cycle

As is shown in Appendix D equation (24) the M.H.D. duct efficiency possible in any particular case may be fixed by defining the ratio of total pressures (or static pressures) across the duct. In practice the duct length required to achieve this efficiency might be too long if the temperature level or magnetic flux strength in the duct is limited. However, in order to obtain an idea of the relationship of overall cycle efficiency to duct efficiency it will be assumed that the duct efficiency defined by the ratio of total pressures across the duct is realistic.

When this model of an M.H.D. duct system is integrated into the full thermal cycle the overall thermal efficiency is given by equation (28) in Appendix E as:

$$\eta_{\substack{th \\ \text{without regen.}}} = 1 - \frac{h_{t_o}}{q}\left\{(1-\eta_{duct})^{1+\frac{\beta}{\gamma}-\beta}\, e^{\frac{\Delta s}{C_p}} - 1\right\} .$$

This equation illustrates that for a fixed amount of heat addition by combustion, q, the overall thermal efficiency decreases as entropy increases due to less favourable choice for the combustion process.

When the type of combustion process is fixed, the relationship of overall thermal efficiency to M.H.D. duct efficiency for various total amounts of heat addition during combustion has also been studied in Appendix E. Explicit optimisation was not found to be possible for most of the combustion processes considered, but for the case of constant pressure (velocity) combustion at low subsonic speeds it was found that the optimum value for the thermal efficiency

is obtained by having the largest amount of heat addition possible during the combustion process. Further the same trend is true for this case at higher flow velocities (given by the velocity parameter $\left(\frac{V_A^2}{2\, h_{t_o}}\right)$) in the combustion chamber, as can be seen from the examples illustrated in Figs.7(a), (b) and (c). Here the cycle efficiency is plotted against the combustion total-heat-addition parameter $\left(\frac{q}{h_{t_o}}\right)$ and is shown by solid curves for each value of the velocity parameter. The regions in these figures for fully subsonic, fully supersonic, and part subsonic and part supersonic combustion are illustrated by the dashed lines. (The Mach numbers assumed for the M.H.D. duct in these examples were 1.4, 0.5 and 2.) Thus it is seen that the M.H.D. duct efficiency of 0.25 is approached only for the case of subsonic combustion with high heat addition, the supersonic combustion region being much less efficient due to the large initial compression which is required to complete the cycle for this type of combustion process.

6 CONCLUDING REMARKS

In the above simplified examination of parts of the M.H.D. power generation cycle no allowances for heat loss or real gas effects have been made in the analysis. Although it might be argued that efficient regeneration processes should compensate to some extent for heat losses, the design of a system which requires the minimum amount of regeneration in this way might well prove to be necessary in practice.

Thus for the operation of an M.H.D. power generation duct of supersonic speeds, the possibility of having an effectively 'cold' nozzle, perhaps with regeneration as Thring suggests, followed by supersonic combustion might be desirable, since then no hot nozzle expansion is necessary, the ambient temperature is limited (see section 3), and the maximum total enthalpy and ambient temperature are produced only at the entry to the M.H.D. duct. However, as is shown in section 4.2, pressure losses may prove to be too high in such a system, and because of the large initial compression required to compensate for this the overall efficiency may be low, as was indicated from the examples studied for supersonic combustion in section 5.

Since, on the other hand, complete combustion before expansion in a nozzle to the required conditions for the M.H.D. duct produces problems of

heat losses and high ambient temperature, a compromise between these two systems is suggested whereby the nozzle and combustion chamber are combined. Thus a 'heat-addition' nozzle would seem appropriate, which would consist of subsonic and supersonic regions of combustion, with the necessary area change to avoid choking in the channel. In fact, combining the roles of the various parts of the system in this way also suggests that further combustion in the M.H.D. duct might prove efficient as a means of 'topping up' the total enthalpy and improving the distribution of power density along the duct.

Appendix A

TYPICAL RELATIONSHIPS BETWEEN THE CYCLE EFFICIENCIES

In section 2.1 we have:

$$\eta_{\substack{th \\ \text{without regen.}}} = \left(\frac{h_{t_o} + w + q}{q}\right) \eta_{duct} - \left(\frac{w}{q}\right) \qquad (5)$$

$$= A\,\eta_{duct} - B \quad , \quad \text{say} \quad .$$

For typical cycles shown in Figs.1 and 2 a rough estimate of the order of magnitude generally quoted for each of the quantities shown is:

$$h_{t_o} \sim 300^\circ K$$

$$q_C = 5\,h_{t_o} \qquad (\sim 1500^\circ K)$$

$$w = \left[(C)^{\frac{\gamma-1}{\gamma}} - 1\right] h_{t_o} \quad ,$$

where C is the initial compression ratio from state 0 to state 1.

$$q_R = 4\,h_{t_o} - (C)^{\frac{\gamma-1}{\gamma}}\,h_{t_o} \quad ,$$

(based on the fact that the limit of regeneration is given by: $h_{t_2} = h_{t_o} + w + q_R = 4\,h_{t_o} \sim 1200^\circ K$).

Hence:

$$q = q_R + q_C = 9\,h_{t_o} - (C)^{\frac{\gamma-1}{\gamma}}\,h_{t_o}$$

$$A = \frac{9}{9 - (C)^{\frac{\gamma-1}{\gamma}}}$$

and

$$B = \frac{(C)^{\frac{\gamma-1}{\gamma}} - 1}{9 - (C)^{\frac{\gamma-1}{\gamma}}} .$$

With $\gamma < 1.3$ then $\frac{\gamma-1}{\gamma} < 0.25$, and if we consider the moderate range for the initial compression ratio of $1 < C < 16$, then:

$$1 < (C)^{\frac{\gamma-1}{\gamma}} < 2 .$$

Therefore for C in this range we have:

$$1.125 < A < 1.285$$

and

$$0 < B < 0.143 .$$

Thus as C increases from 1 to 16 the corresponding change in equation (5) is given by:

$$\eta_{\substack{th \\ \text{without regen.}}} = [1.125 \text{ to } 1.285]\ \eta_{duct} - [0 \text{ to } 0.143] .$$

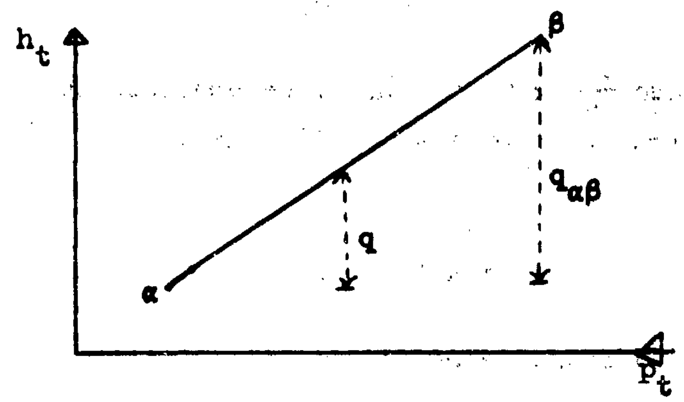
In addition the conversion factor given in (3) for conversion to the case with regeneration is given by:

$$\frac{q}{q_C} = \frac{9 - (C)^{\frac{\gamma-1}{\gamma}}}{5} ,$$

and this decreases from 1.6 to 1.4 as C increases from 1 to 16.

Appendix B

SUMMARY OF THE RELATIONSHIPS BETWEEN GAS PROPERTIES DURING COMBUSTION IN OR HEAT ADDITION TO THE FLOW



For a typical leg of the cycle α-β, where an amount of heat, q, is either added to or produced by combustion in the flow, we have the following relations when an ideal gas-model is assumed:

$$q = h_t - h_{t_\alpha}$$

or

$$dq = d\,h_t \qquad \text{Energy equation (8)}$$

$$dp = -\rho v\, dv \qquad \text{Momentum equation (9)}$$

$$p = \left(\frac{\gamma - 1}{\gamma}\right) \rho h \qquad \text{Equation of state (10)}$$

$$h_t = h + \tfrac{1}{2} v^2 \qquad \text{Definition of } h_t \text{ (11)}$$

$$\frac{p_t}{p} = \left(\frac{h_t}{h}\right)^{\frac{\gamma}{(\gamma-1)}} \left[= \left(\frac{T_t}{T}\right)^{\frac{\gamma}{\gamma-1}} \right] \qquad \text{Definition of } p_t \text{ (12)}$$

From (9), (10) and (11) we have:

$$\frac{dp}{p} = -\left(\frac{\rho}{p}\right)(v\, dv) = -\frac{\gamma}{(\gamma - 1)} \left\{\frac{d\,h_t - dh}{h}\right\} \qquad (13)$$

By logarithmic differentiation of (12) we get:

$$\frac{d\,p_t}{p_t} - \frac{dp}{p} = \frac{\gamma}{(\gamma - 1)}\left\{\frac{d\,h_t}{h_t} - \frac{dh}{h}\right\} \quad . \tag{14}$$

Hence by adding (13) and (14) we obtain:

$$\frac{d\,p_t}{p_t} = \frac{\gamma}{(\gamma - 1)}\left\{\frac{1}{h_t} - \frac{1}{h}\right\} d\,h_t = \frac{\gamma}{(\gamma - 1)}\left\{\frac{1}{(q + h_{t_\alpha})} - \frac{1}{h}\right\} dq \quad , \text{ from (8)}$$

$$\ldots \quad (15)$$

Integration of (15) gives:

$$\left(\frac{p_{t_\beta}}{p_{t_\alpha}}\right)^{\frac{(\gamma-1)}{\gamma}} = \left\{\frac{q_{\alpha\beta} + h_{t_\alpha}}{h_{t_\alpha}}\right\} e^{-\int_0^{q_{\alpha\beta}} \frac{dq}{h}} \tag{16}$$

(where in terms of the specific heat we have:

$$\int_0^{q_{\alpha\beta}} \frac{dq}{h} = \frac{\Delta s}{C_p} = \frac{s_\beta - s_\alpha}{C_p}\Big) \quad .$$

From (8) we have:

$$h_{t_\beta} = h_{t_\alpha} + q_{\alpha\beta} \quad . \tag{17}$$

Equations (16) and (17) thus give the total pressure and total enthalpy at β. The determination of the integral in (16), however, cannot be made until the combustion or heat addition process between α and β has been defined.

Particular examples for the combustion (heat addition) process which are usually considered are:

(i) Constant static enthalpy.

(ii) Constant static pressure (i.e. constant velocity).

(iii) Constant Mach number.

(iv) Constant area.

For these processes the formulae required for the determination of the properties at β in addition to equations (8) to (17) are given below.

(i) <u>Constant static enthalpy</u>

$$h_\beta = h_\alpha \ (= h) ,$$

and so we have

$$\int_0^{q_{\alpha\beta}} \frac{dq}{h} = \frac{q_{\alpha\beta}}{h_\alpha} .$$

(ii) <u>Constant static pressure (velocity)</u>

$$p_\beta = p_\alpha .$$

$$v_\beta = v_\alpha .$$

And

$$\int_0^{q_{\alpha\beta}} \frac{dq}{h} = \int_0^{q_{\alpha\beta}} \frac{dq}{(q + h_{t_\alpha} - \frac{1}{2} v_\alpha^2)} = \log_e \left(\frac{q_{\alpha\beta} + h_{t_\alpha} - \frac{1}{2} v_\alpha^2}{h_{t_\alpha} - \frac{1}{2} v_\alpha^2} \right) .$$

(iii) <u>Constant Mach number</u>

$\frac{h_t}{h} = 1 + \frac{1}{2} (\gamma - 1) M^2 =$ const., at all points from α to β.

$$\left(\text{Note: } \frac{h_\beta}{h_\alpha} = \frac{h_{t_\beta}}{h_{t_\alpha}} .\right)$$

Therefore:

$$\int_0^{q_{\alpha\beta}} \frac{dq}{h} = (1 + \tfrac{1}{2}(\gamma - 1) M^2) \int_0^{q_{\alpha\beta}} \frac{dq}{h_t}$$

$$= (1 + \tfrac{1}{2}(\gamma - 1) M^2) \int_0^{q_{\alpha\beta}} \frac{dq}{(q + h_{t_\alpha})}$$

$$= (1 + \tfrac{1}{2}(\gamma - 1) M^2) \log_e \left(\frac{q_{\alpha\beta} + h_{t_\alpha}}{h_{t_\alpha}}\right)$$

$$= \frac{h_{t_\alpha}}{h_\alpha} \log_e \left(\frac{q_{\alpha\beta} + h_{t_\alpha}}{h_{t_\alpha}}\right) .$$

(iv) <u>Constant area</u>

$$\rho_\beta v_\beta = \rho_\alpha v_\alpha \; (= \rho v) \quad , \quad \text{continuity of mass.} \tag{18}$$

Using this equation (9) integrates to give:

$$p + \rho v^2 = \text{const.}$$

i.e.

$$p_\beta + \rho_\beta v_\beta^2 = p_\alpha + \rho_\alpha v_\alpha^2 \; (= p + \rho v^2) \quad . \tag{19}$$

From (18), (19), (8) and (11) we have that for a general point:

$$\left.\begin{aligned} \rho &= \frac{\rho_\alpha v_\alpha}{v} \\ p &= (p_\alpha + \rho_\alpha v_\alpha^2) - (\rho_\alpha v_\alpha) v \\ h &= (h_{t_\alpha} + q) - \tfrac{1}{2} v^2 \end{aligned}\right\} \quad . \tag{20}$$

Using (10) and (20) we obtain a quadratic for v:

$$(\gamma + 1)\ v^2 - 2\gamma\ v_\alpha \left(\frac{p_\alpha}{\rho_\alpha\ v_\alpha^2} + 1 \right) v + 2\ (\gamma - 1)\ (h_{t_\alpha} + q) = 0\ .$$

The realistic solution for v from this quadratic is:

$$v = \frac{\gamma\ v_\alpha \left(\frac{p_\alpha}{\rho_\alpha\ v_\alpha^2} + 1 \right) + \sqrt{\gamma^2\ v_\alpha^2 \left(\frac{p_\alpha}{\rho_\alpha\ v_\alpha^2} + 1 \right)^2 - 2(\gamma^2 - 1)\ (h_{t_\alpha} + q)}}{(\gamma + 1)}\ . \qquad \ldots (21)$$

Thus using (21) together with (20), (11), (12) and (17) the properties at point β may be determined. It should be noted that in this case it is not convenient to calculate $\int_0^{q_{\alpha\beta}} \frac{dq}{h}$ in order to determine p_{t_β} directly. Note from (8), (11), (12) and (20) we have:

$$\begin{aligned} p_t &= p\left(\frac{h_t}{h}\right)^{\frac{\gamma}{(\gamma-1)}} \\ &= \left[(p_\alpha + \rho_\alpha\ v_\alpha^2) - (\rho_\alpha\ v_\alpha)\ v\right] \left(\frac{h_{t_\alpha} + q}{h_{t_\alpha} + q - \frac{1}{2}\ v^2}\right)^{\frac{\gamma}{(\gamma-1)}}\ . \end{aligned}$$

Appendix C

FORMULAE RELATING THE VARIOUS PARTS OF THE MODEL FOR THE COMBUSTION-PLUS-NOZZLES SECTION OF THE CYCLE

(see Fig. 3)

(i) The process from 1 (or 2) to A in Fig. 3 is an isentropic nozzle expansion and is given by:

$$h_{t_A} = h_{t_1} \qquad (\text{or } h_{t_2})$$

$$p_{t_A} = p_{t_1} \qquad (\text{or } p_{t_2})$$

v_A ($< (2\, h_{t_A})^{\frac{1}{2}}$) depends upon the nozzle expansion, and can therefore be chosen arbitrarily.

(ii) Process from A to B is given by: ($h_B \equiv h_{max}$)

$$h_{t_B} = h_{t_A} + \lambda q$$

together with (for the following special cases given in Appendix B):

(a) Constant static pressure (velocity)

$$v_B = v_A$$

and

$$\left(\frac{p_{t_B}}{p_{t_A}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{h_{t_A} + \lambda q}{h_{t_A}}\right)\left(\frac{h_{t_A} - \frac{1}{2} v_A^2}{h_{t_A} + \lambda q - \frac{1}{2} v_A^2}\right)$$

$$(\text{or } p_B = p_A)$$

(b) Constant Mach number

$$v_B = \sqrt{\left(\frac{h_{t_A} + \lambda q}{h_{t_A}}\right)} \cdot v_A$$

and

$$\left(\frac{p_{t_B}}{p_{t_A}}\right)^{\frac{\gamma-1}{\gamma}} = e^{-\frac{h_{t_A}}{h_A}} = e^{-\frac{h_{t_A}}{(h_{t_A} - \frac{1}{2} v_A^2)}} .$$

(c) Constant area

$$v_B = \frac{\gamma\, v_A \left(\frac{p_A}{\rho_A\, v_A^2} + 1\right) + \sqrt{\gamma^2\, v_A^2 \left(\frac{p_A}{\rho_A\, v_A^2} + 1\right)^2 - 2(\gamma^2 - 1)\,(h_{t_A} + \lambda q)}}{(\gamma + 1)}$$

and

$$p_{t_B} = \left[(p_A + \rho_A\, v_A^2) - (\rho_A\, v_A)\, v_B\right] \left(\frac{h_{t_A} + \lambda q}{h_{t_A} + \lambda q - \frac{1}{2} v_B^2}\right)^{\frac{\gamma}{(\gamma-1)}} .$$

(iii) Process from B to D is given by: $(h_D \equiv h_{max})$

$$h_{t_D} = h_{t_B} + (1 - \lambda)\, q$$

$$v_D = \sqrt{2\,(h_{t_D} - h_B)}$$

$$(\text{or } h_D = h_B)$$

and from the constant ambient enthalpy process given in Appendix B

$$\left(\frac{p_{t_D}}{p_{t_B}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{h_{t_B} + (1 - \lambda)\, q}{h_{t_B}}\right) e^{-\frac{(1-\lambda)\, q}{h_B}} .$$

(iv) Process from D to 4 is again an isentropic nozzle expansion given by:

$$h_{t_4} = h_{t_D}$$

$$p_{t_4} = p_{t_D}$$

with v_4 $(< \sqrt{2\, h_{t_4}})$ chosen to give required velocity at state 4.

Appendix D

FORMULAE RELATING PROPERTIES IN A CONSTANT MACH NUMBER M.H.D. DUCT

From Ref.6 using the notation of Fig.2 we find that for the constant Mach number M.H.D. duct the efficiency of the duct, η_{duct}, and temperature and pressure ratios across it are given by:

$$\eta_{duct} = 1 - \frac{T_{t_5}}{T_{t_4}} \tag{22}$$

and

$$\frac{T_{t_5}}{T_{t_4}} = \frac{T_5}{T_4} = \left(\frac{p_5}{p_4}\right)^{\frac{1}{\beta}} = \left(\frac{p_{t_5}}{p_{t_4}}\right)^{\frac{1}{\beta}} \tag{23}$$

where $\beta = \{1 + \frac{1}{2}(1-K)(\gamma-1) M_4^2\} \gamma/(\gamma-1) K$, and M_4 and γ are constant.

K is considered constant in this analysis ($0 < K < 1$) and is the external load factor $\left(= \frac{E}{vB} \text{ (see list of symbols)}\right)$ at each point in the duct. A typical value for K being 0.5 to 0.75 for an efficient duct.

From (22) and (23) we obtain:

$$\frac{p_{t_4}}{p_{t_5}} = (1 - \eta_{duct})^{-\beta} \quad . \tag{24}$$

This relationship is illustrated in Fig.6 and shows how the ratio of total pressure across the M.H.D. duct varies with the efficiency of the duct, for $\gamma = 1.3$ and various values of M_4 and K.

Although for particular values of M_4, γ and K the duct efficiency depends only on pressure and temperature ratios across the duct, the duct length required to achieve this efficiency depends on the absolute values of the conditions in the duct. It must always be remembered, therefore, when a value for the duct efficiency is assumed, that in practice this value may correspond to an unrealistic length for the duct, when, for example, the temperature level or magnetic flux strength is limited. Also it should be

noted that optimum duct efficiency does not necessarily correspond to optimum overall efficiency for the full thermal cycle.

For a constant Mach number duct with constant K and γ the duct length x is given by:

$$\frac{x}{x^*} = 1 - (1 - \eta_{duct})^{w'} \qquad (25)$$

where $w' = (\beta - \frac{1}{2}) - (y' - \beta z')$,

$$x^* = \frac{(\rho_4 \ v_4) \ h_{t_4}}{K(1 - K) \ w' \cdot \sigma_4 \ V_4^2 \ B^2} \ ,$$

and the indices y' and z' originate from an expression of the form $\sigma \propto T^{y'} \ p^{-z'}$ for the electrical conductivity σ. Typically y' is of the order of 10-13 and z' is of the order of 0.4-0.5.

Appendix E

INTEGRATION OF THE CONSTANT MACH NUMBER M.H.D. DUCT INTO THE COMPLETE CYCLE

Using the notation in Figs. 2 and 3, from Appendix B equation (16) we have directly, by considering the energy addition w during isentropic compression and heat addition q during combustion:

$$\left(\frac{p_{t_4}}{p_{t_0}}\right)^{\frac{\gamma-1}{\gamma}} = \left\{\frac{q + w + h_{t_0}}{h_{t_0}}\right\} e^{-\int_0^q \frac{dq}{h}}$$

$$= \left\{\frac{q + w + h_{t_0}}{h_{t_0}}\right\} e^{-\frac{\Delta s}{C_p}}$$

where Δs is the entropy gain in the process.

If we assume that $p_{t_5} = p_{t_0}$ (i.e. that the duct operates down to the initial pressure), then by matching the pressure through the system we see that:

$$\left(\frac{p_{t_4}}{p_{t_5}}\right)^{\frac{\gamma-1}{\gamma}} = \left\{\frac{q + w + h_{t_0}}{h_{t_0}}\right\} e^{-\frac{\Delta s}{C_p}} . \tag{26}$$

From equation (26) and (24) in Appendix D it follows that:

$$\left.\begin{aligned} w &= h_{t_0}\left\{\left(\frac{p_{t_4}}{p_{t_5}}\right)^{\frac{\gamma-1}{\gamma}} e^{\frac{\Delta s}{C_p}} - 1\right\} - q \\ &= h_{t_0}\left\{(1 - \eta_{duct})^{-\frac{\beta}{\gamma}(\gamma-1)} e^{\frac{\Delta s}{C_p}} - 1\right\} - q \end{aligned}\right\} . \tag{27}$$

This shows how an increase in initial compression, given by the energy parameter w, arises from an increase in entropy in the system.

From (5) section 2.1, (24) Appendix D, and (27) eliminating w we get:

$$\left.\begin{aligned}\eta_{th\ \text{without regen.}} &= 1 - \frac{h_{t_o}}{q}\left\{\left(\frac{p_{t_4}}{p_{t_5}}\right)^{\left(1-\frac{1}{\gamma}-\frac{1}{\beta}\right)} e^{\frac{\Delta s}{C_p}} - 1\right\} \\ &= 1 - \frac{h_{t_o}}{q}\left\{(1-\eta_{duct})^{\left(1+\frac{\beta}{\gamma}-\beta\right)} e^{\frac{\Delta s}{C_p}} - 1\right\}\end{aligned}\right\} . \qquad (28)$$

Equation (28) illustrates how for a fixed amount of heat addition, q, the thermal efficiency of the cycle decreases as entropy in the system increases. Hence the most efficient system for a fixed value of q requires the minimum entropy gain or total pressure loss in the combustion part of the cycle.

This is an obvious result and is associated with the comparison of various combustion processes for a fixed amount of heat addition, q. However, for a particular combustion process, optimisation of the thermal efficiency when q is varied is far more complex, since in (28) Δs is a function of q.

Differentiating (28) with respect to q it follows that for stationary values of $\eta_{th\ \text{without regen.}}$, q must satisfy the equation:

$$\frac{d}{dq}\left\{\frac{e^{\frac{\Delta s}{C_p}}}{q}\right\} = -\frac{1}{q^2}(1-\eta_{duct})^{\left(\beta-1-\frac{\beta}{\gamma}\right)}$$

or

$$e^{\frac{\Delta s}{C_p}}\left(1 - q\frac{d\left(\frac{\Delta s}{C_p}\right)}{dq}\right) = (1-\eta_{duct})^{\left(\beta-1-\frac{\beta}{\gamma}\right)} . \qquad (29)$$

Hence if a clear maximum value of $\eta_{th\ \text{without regen.}}$ exists it will correspond to a value for q given by (29). In (29) Δs may be assumed to be a function of q and is found from (27), assuming w also as a function of q, and the appropriate law for the combustion process assumed.

As examples of this optimisation process we may consider in detail the cases of (i) constant static enthalpy, (ii) constant static pressure (velocity), and (iii) constant Mach number, combustion, which are listed in Appendix B. For these processes we may consider combustion, using the notation of Figs. 1 and 2, to start from some state A between 1 (or 2) and 4 as in Fig. 3.

Hence we have in addition to equation (27) for each case the following equations from Appendix B:

(i)
$$\frac{\Delta s}{C_p} = \frac{q}{h_A} = \frac{q}{(w + h_{t_o} - \frac{1}{2} v_A^2)}$$

(ii)
$$e^{\frac{\Delta s}{C_p}} = \frac{(q + h_{t_A} - \frac{1}{2} v_A^2)}{(h_{t_A} - \frac{1}{2} v_A^2)} = \frac{(q + w + h_{t_o} - \frac{1}{2} v_A^2)}{(w + h_{t_o} - \frac{1}{2} v_A^2)}$$

(iii)
$$\left.\begin{aligned} e^{\frac{\Delta s}{C_p}} &= \left(\frac{q + h_{t_A}}{h_{t_A}}\right) e^{\frac{h_{t_A}}{h_A}} \\ &= \left(\frac{q + w + h_{t_o}}{w + h_{t_o}}\right) e^{\left\{\frac{w + h_{t_o}}{w + h_{t_o} - \frac{1}{2} v_A^2}\right\}} \\ &= \left(\frac{q + w + h_{t_o}}{w + h_{t_o}}\right) e^{\{1 + \frac{1}{2}(\gamma - 1) M_A^2\}} \end{aligned}\right\} \qquad (30)$$

Particular case:

The simplest case above to study algebraically is that of constant static pressure combustion. For this case equations (27) and (30) (ii) may be solved for $e^{\frac{\Delta s}{C_p}}$ and w directly to give:

$$e^{\frac{\Delta s}{C_p}} = 1 + \frac{2\left(\frac{q}{h_{t_o}}\right)}{\left\{(1-\eta_{duct})^{-\frac{\beta}{\gamma}(\gamma-1)} - \left(\frac{v_A^2}{2\,h_{t_o}}\right) - \left(\frac{q}{h_{t_o}}\right) + \theta\right\}} \qquad (31)$$

$$w = \frac{h_{t_o}}{2}\left\{(1-\eta_{duct})^{-\frac{\beta}{\gamma}(\gamma-1)} - 2 + \left(\frac{v_A^2}{2\,h_{t_o}}\right) - \left(\frac{q}{h_{t_o}}\right) + \theta\right\} \qquad (32)$$

where

$$\theta = \left\{\left[\frac{q}{h_{t_o}} + \left(\frac{v_A^2}{2\,h_{t_o}}\right) + (1-\eta_{duct})^{-\frac{\beta}{\gamma}(\gamma-1)}\right]^2 - 4\left(\frac{v_A^2}{2\,h_{t_o}}\right)(1-\eta_{duct})^{-\frac{\beta}{\gamma}(\gamma-1)}\right\}^{\frac{1}{2}}$$

However, substitution of $e^{\frac{\Delta s}{C_p}}$ from equation (31) into (29) results in an equation for q which is not amenable to simple analytical treatment. Hence it is better to study the behaviour of the thermal efficiency with q more directly by substituting for $e^{\frac{\Delta s}{C_p}}$ from (31) into equation (28). This gives:

$$\eta_{th \text{ without regen.}} = 1 - \frac{1}{\left(\frac{q}{h_{t_o}}\right)}\left\{(1-\eta_{duct})^{(1+\frac{\beta}{\gamma}-\beta)} - 1 + \frac{2\left(\frac{q}{h_{t_o}}\right)(1-\eta_{duct})^{1+\frac{\beta}{\gamma}-\beta}}{\left[(1-\eta_{duct})^{-\frac{\beta}{\gamma}(\gamma-1)} - \left(\frac{v_A^2}{2\,h_{t_o}}\right) - \left(\frac{q}{h_{t_o}}\right) + \theta\right]}\right\} \qquad \ldots (33)$$

At any stage during the combustion process we have:

$$v_A^2 = (\gamma - 1)\, M^2\, h = (\gamma - 1)\, M^2 \left(w + h_{t_o} + q' - \tfrac{1}{2} v_A^2\right) \qquad (34)$$

where q' is the amount of heat added up to that stage during combustion and $0 \leqslant q' \leqslant q$.

Hence, from (34), the flow during combustion will be sonic or supersonic at points where:

$$v_A^2 \geqslant (\gamma - 1)\left(w + h_{t_o} + q' - \tfrac{1}{2} v_A^2\right)$$

i.e.

$$\frac{1}{2}\left(\frac{\gamma + 1}{\gamma - 1}\right) v_A^2 \geqslant \left(w + h_{t_o} + q'\right) \quad . \qquad (35)$$

Thus since q' lies in the range $0 \leqslant q' \leqslant q$, the combustion process will be completely supersonic when:

$$\frac{1}{2}\left(\frac{\gamma + 1}{\gamma - 1}\right) v_A^2 > \left(w + h_{t_o} + q\right)$$

and completely subsonic when: (36)

$$\frac{1}{2}\left(\frac{\gamma + 1}{\gamma - 1}\right) v_A^2 < \left(w + h_{t_o}\right)$$

(36) and (32) may therefore be used to define the regions for completely supersonic and completely subsonic combustion. In between we have a region where, in this constant velocity, v_A, case, the flow starts off supersonic but gradually reduces to subsonic as heat is added during combustion.

As a special case of equation (33) we have the case of low subsonic combustion when $\left(\dfrac{v_A^2}{2\, h_{t_o}}\right)$ may be neglected. Hence we get in this case:

$$\eta_{\substack{\text{th}\\ \text{without regen.}}} = \eta_{\text{duct}} - \frac{1}{\left(\dfrac{q}{h_{t_o}}\right)}\left\{\left(1 - \eta_{\text{duct}}\right)^{1 - \frac{\beta(\gamma-1)}{\gamma}} - 1\right\} \quad . \qquad (37)$$

Equation (37) shows that for combustion at low subsonic speeds the maximum value of the thermal efficiency is obtained for the largest value of $\left(\frac{q}{h_{t_o}}\right)$. $\left(\text{Since } \frac{\beta(\gamma-1)}{\gamma} > 1\text{, see (23) Appendix D.}\right)$

In the case of higher flow velocities in the combustion chamber, equation (33) in fact gives a similar trend for the variation of the thermal efficiency with the total amount of heat addition during combustion. This is clearly seen in Fig.7(a), (b) and (c) where cases associated with a constant Mach number M.H.D. duct ($K = 0.75$), for Mach numbers of 1.4, 0.5 and 2 respectively and duct efficiency of 0.25, are plotted for various values of the combustion velocity parameter $\left(\frac{v_A^2}{2\, h_{t_o}}\right)$. Also shown are the regions for fully subsonic and fully supersonic flow during combustion which are given by equations (36) and (32).

SYMBOLS

T	temperature	($^\circ$K)
v	velocity	(m/sec)
h	enthalpy	(m^2/sec^2)
h_t	total enthalpy	(m^2/sec^2)
M	Mach number	
q_R	heat added by regeneration	(m^2/sec^2)
q_C	heat added by combustion	(m^2/sec^2)
q	total heat addition	(m^2/sec^2)
q'	intermediate heat addition	(m^2/sec^2)
$q_{\alpha\beta}$	heat addition on going from state α to state β	(m^2/sec^2)
C_p	specific heat at constant pressure	($m^2/sec^2{}^\circ K$)
s	entropy	($m^2/sec^2{}^\circ K$)
	$\left(\text{Defined by } \frac{\Delta s}{C_p} = \int_0^q \frac{dq'}{h}.\right)$	
w	energy added during initial isentropic compression	(m^2/sec^2)
p	ambient pressure	(N/m^2 or atmospheres)
p_t	total pressure	(N/m^2 or atmospheres)
P	energy output from M.H.D. duct	(m^2/sec^2)
γ	ratio of specific heats	
C	initial compression ratio	
θ	expression defined in Appendix D for equations (31) and (32)	
A	expressions defined in Appendix A	
B	Expression defined in Appendix A	
x	length of M.H.D. duct	(m)
x^*	"characteristic" length for M.H.D. duct given by equation (25) in Appendix D	(m)
y'	index used in Appendix D for σ	
z'	index used in Appendix D for σ	
w'	index used in Appendix D equation (25)	
β	index used in Appendix D equation (23)	
σ	electrical conductivity	
K	external load factor for M.H.D. duct $\left(= \frac{E}{vB}\right)$	

SYMBOLS (Contd.)

E	electric field across the duct	(webers/ m sec = volts/m)
B	magnetic field across the duct	(webers/m^2)

Suffices

t	total conditions
0,1,2,...7	stages in the cycle

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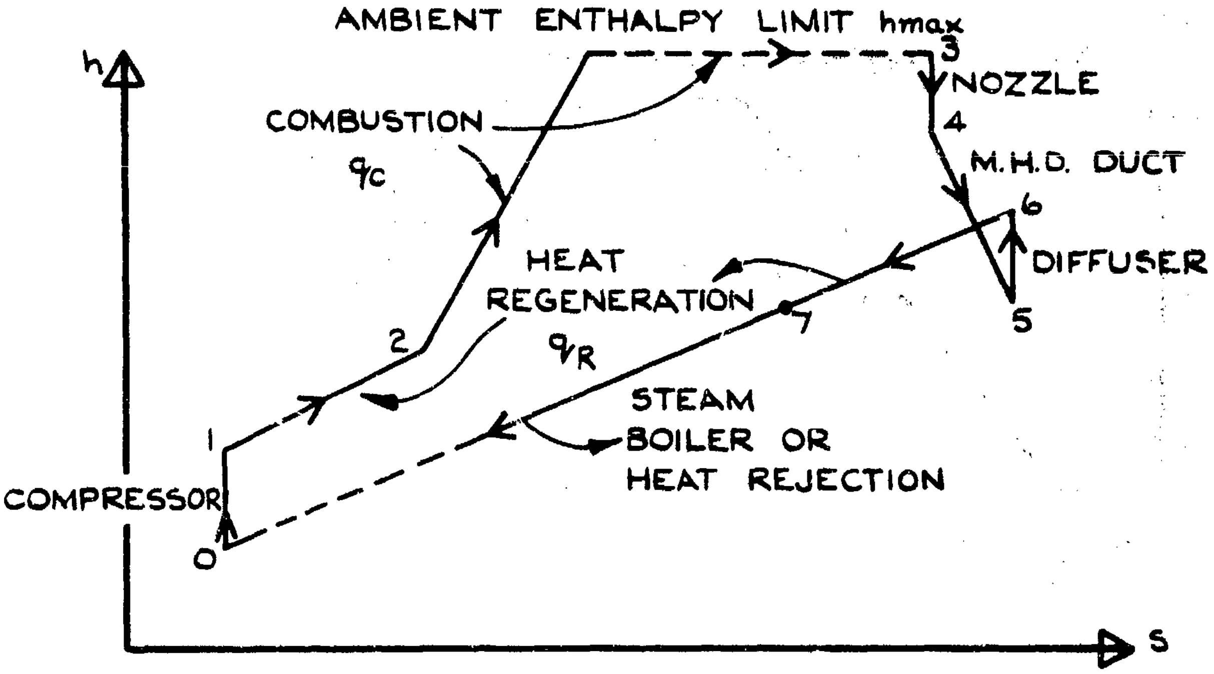
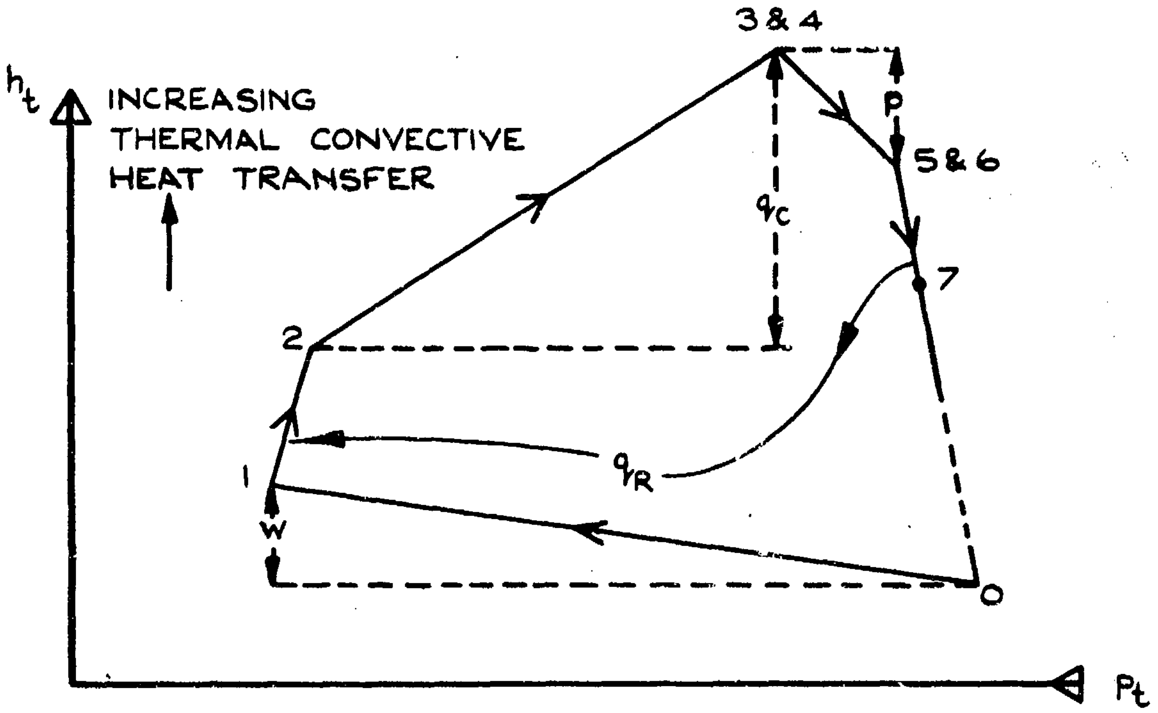


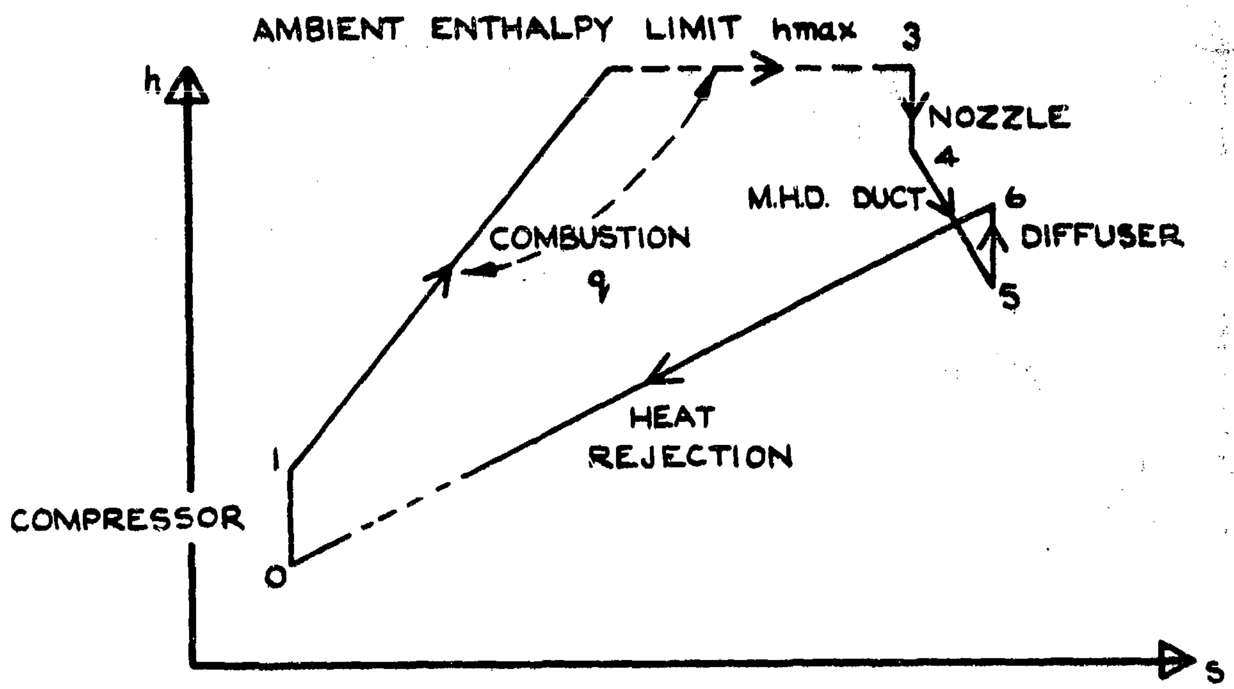
a. h-s DIAGRAM OF THE CYCLE



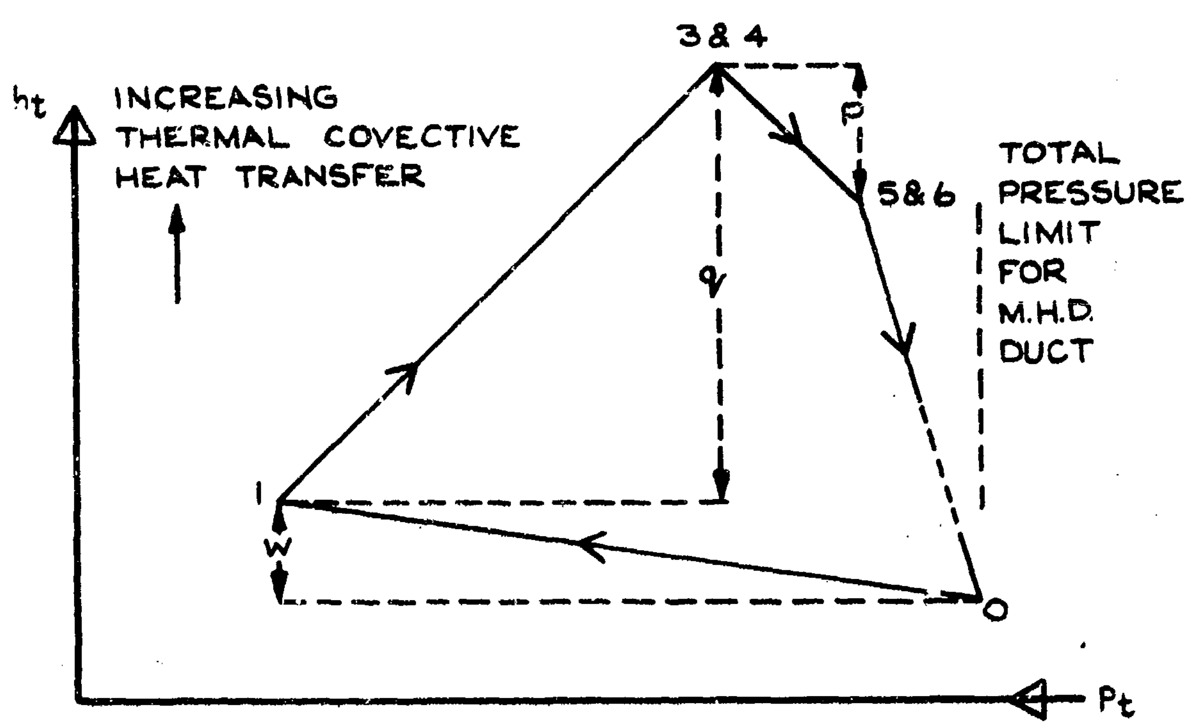
b. $h_t - p_t$ DIAGRAM OF THE CYCLE

FIG. 1 a & b IDEALISED POWER GENERATION CYCLE

Fig. 2 a & b 005 903180



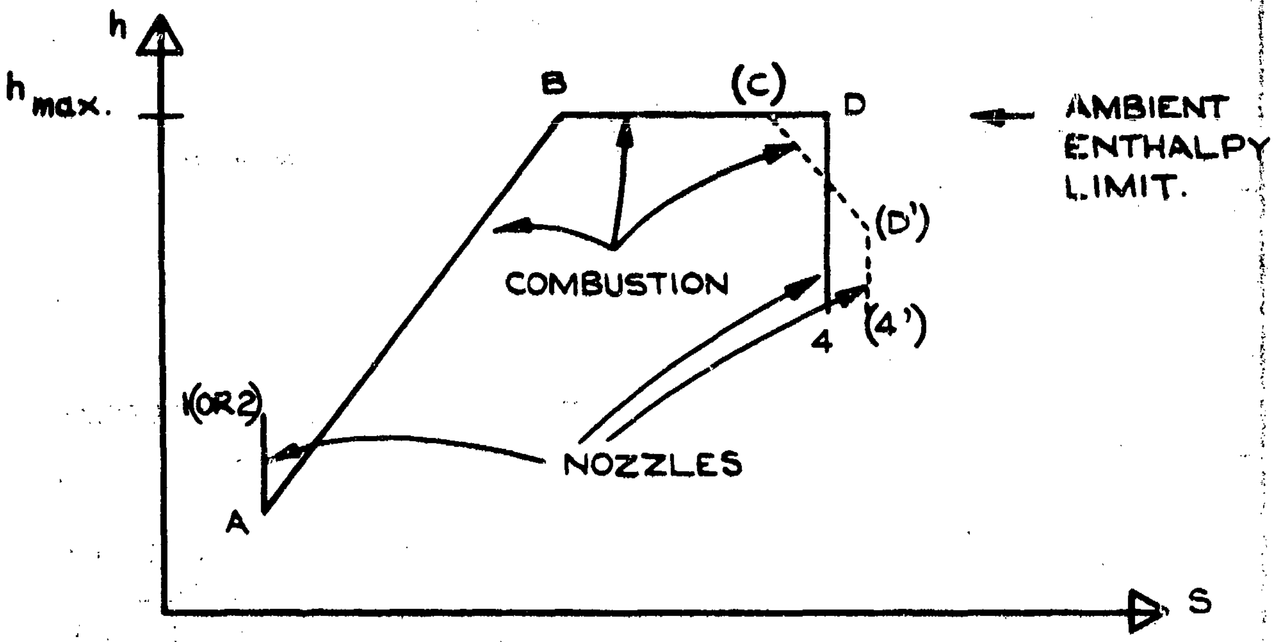
a. h-s DIAGRAM OF THE CYCLE



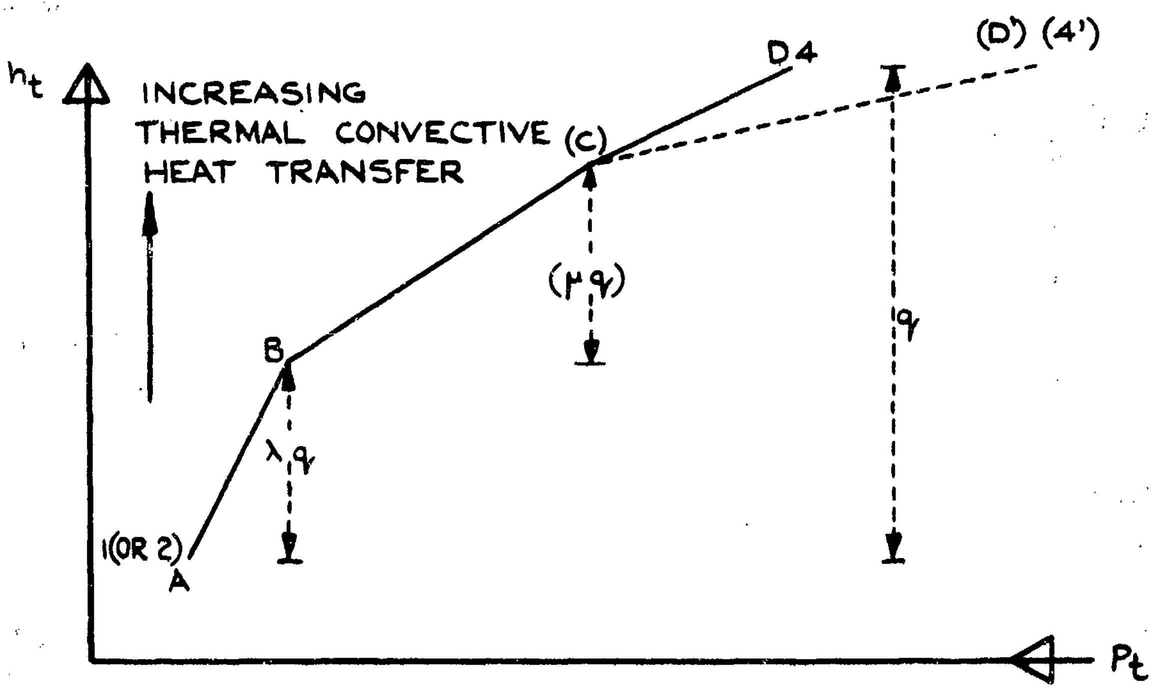
b. $h_t - p_t$ DIAGRAM OF THE CYCLE

FIG. 2 a & b POWER GENERATION CYCLE WITHOUT HEAT REGENERATION

Fig. 3 a&b



a. h – s REPRESENTATION.



b. $h_t - p_t$ REPRESENTATION

FIG. 3 a & b MODEL OF THE PROCESSES WHICH MAY BE CONSIDERED IN THE COMBUSTION - PLUS - NOZZLES SECTION OF THE CYCLE.

Fig. 4 a&b

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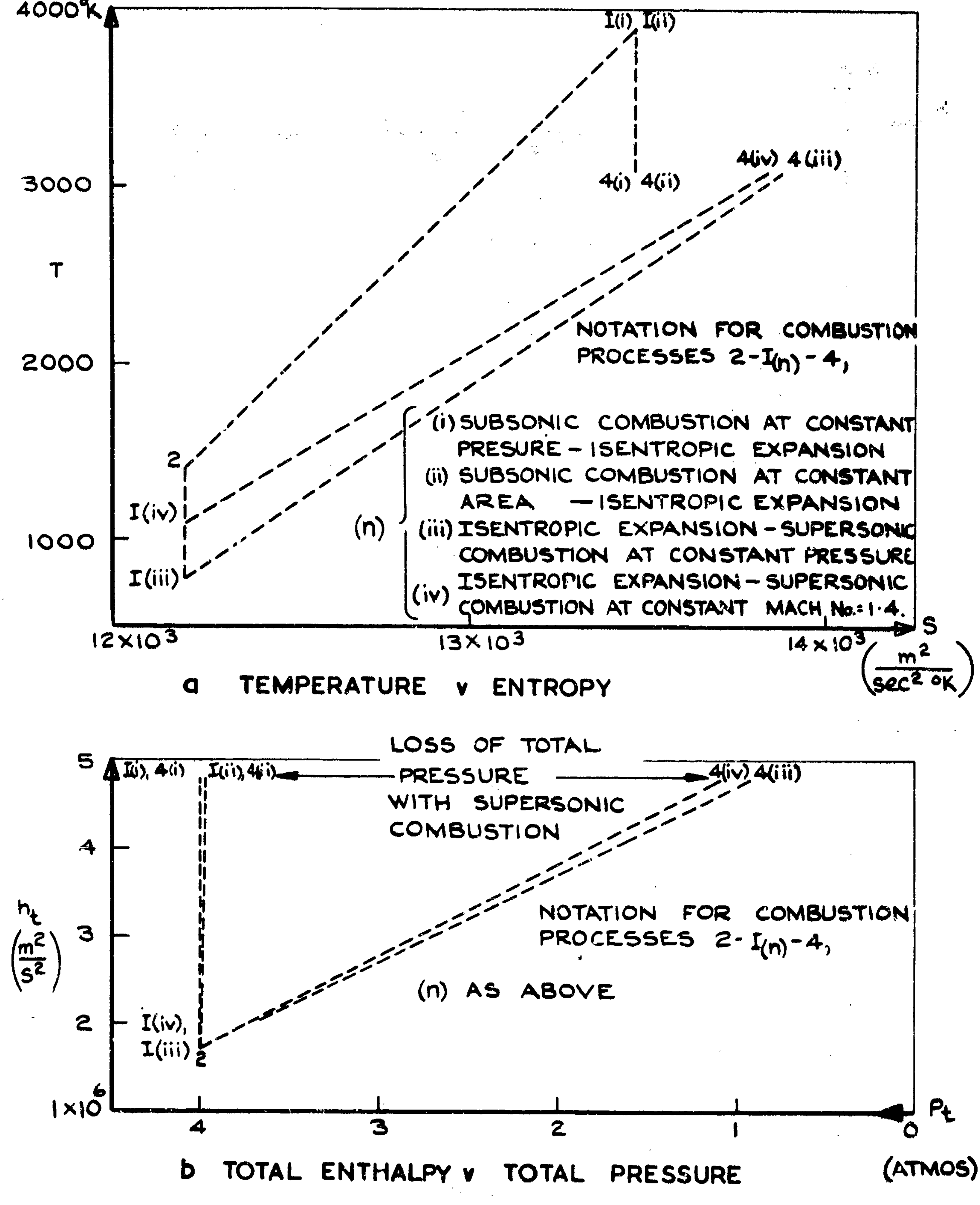
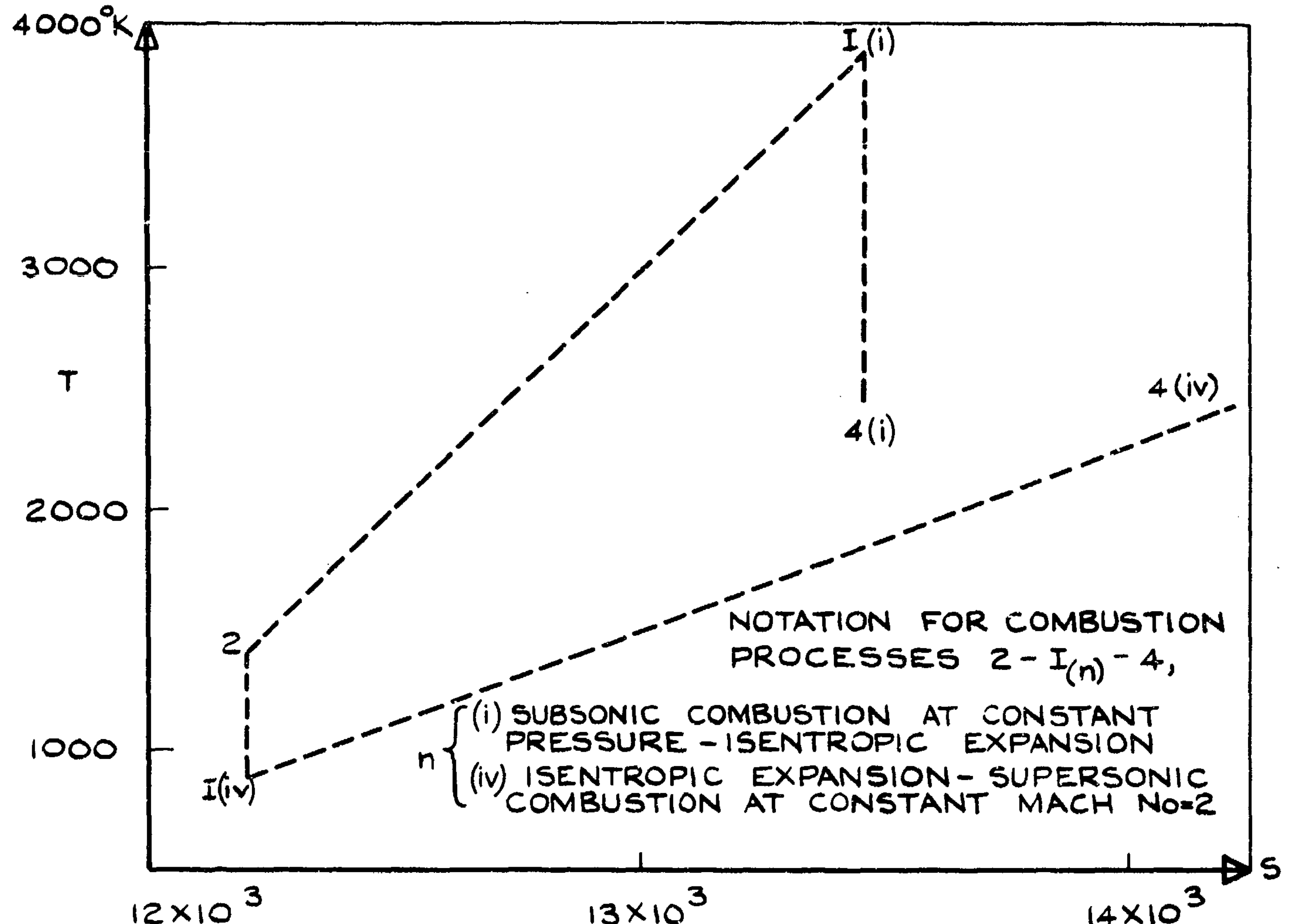
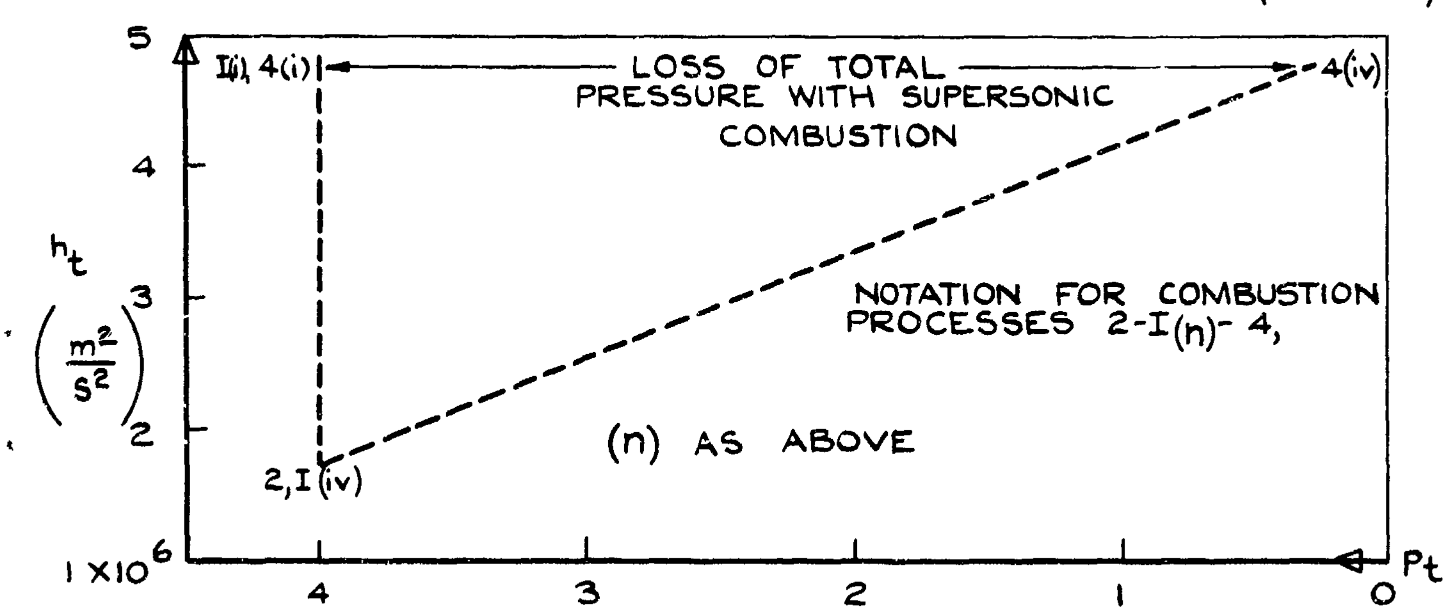


FIG. 4 a&b RESULTS FROM MODEL GIVEN IN FIG. 3, WITH COMPLETELY SUPERSONIC AND COMPLETELY SUBSONIC COMBUSTION PROCESSES. MACH NUMBER AT ENTRY TO M.H.D. DUCT $M_4 = 1 \cdot 4$, $\gamma = 1 \cdot 3$ (AFTER THOMPSON)



a TEMPERATURE v ENTROPY



b TOTAL ENTHALPY v TOTAL PRESSURE

FIG. 5a & b RESULTS FROM MODEL GIVEN IN FIG. 3 WITH COMPLETELY SUPERSONIC AND COMPLETELY SUBSONIC COMBUSTION PROCESSES. MACH NUMBER AT ENTRY TO M.H.D. DUCT $M_4 = 2$, $\gamma = 1{\cdot}3$ (AFTER THOMPSON)

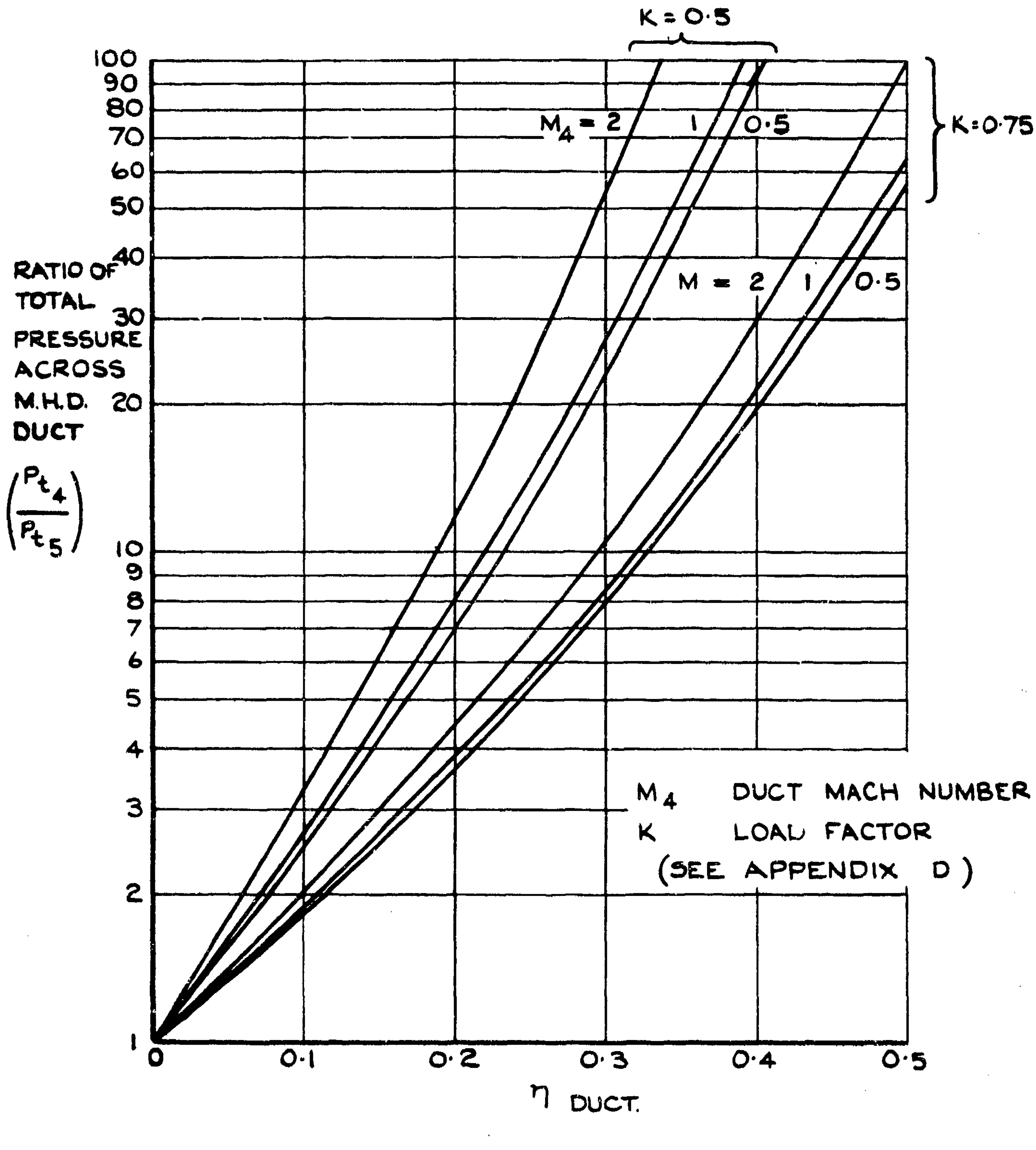


FIG. 6 EFFICIENCY OF A CONSTANT MACH NUMBER M.H.D. DUCT VERSUS RATIO OF TOTAL PRESSURE ACROSS THE DUCT ($\gamma = 1{\cdot}3$)

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Fig. 7a

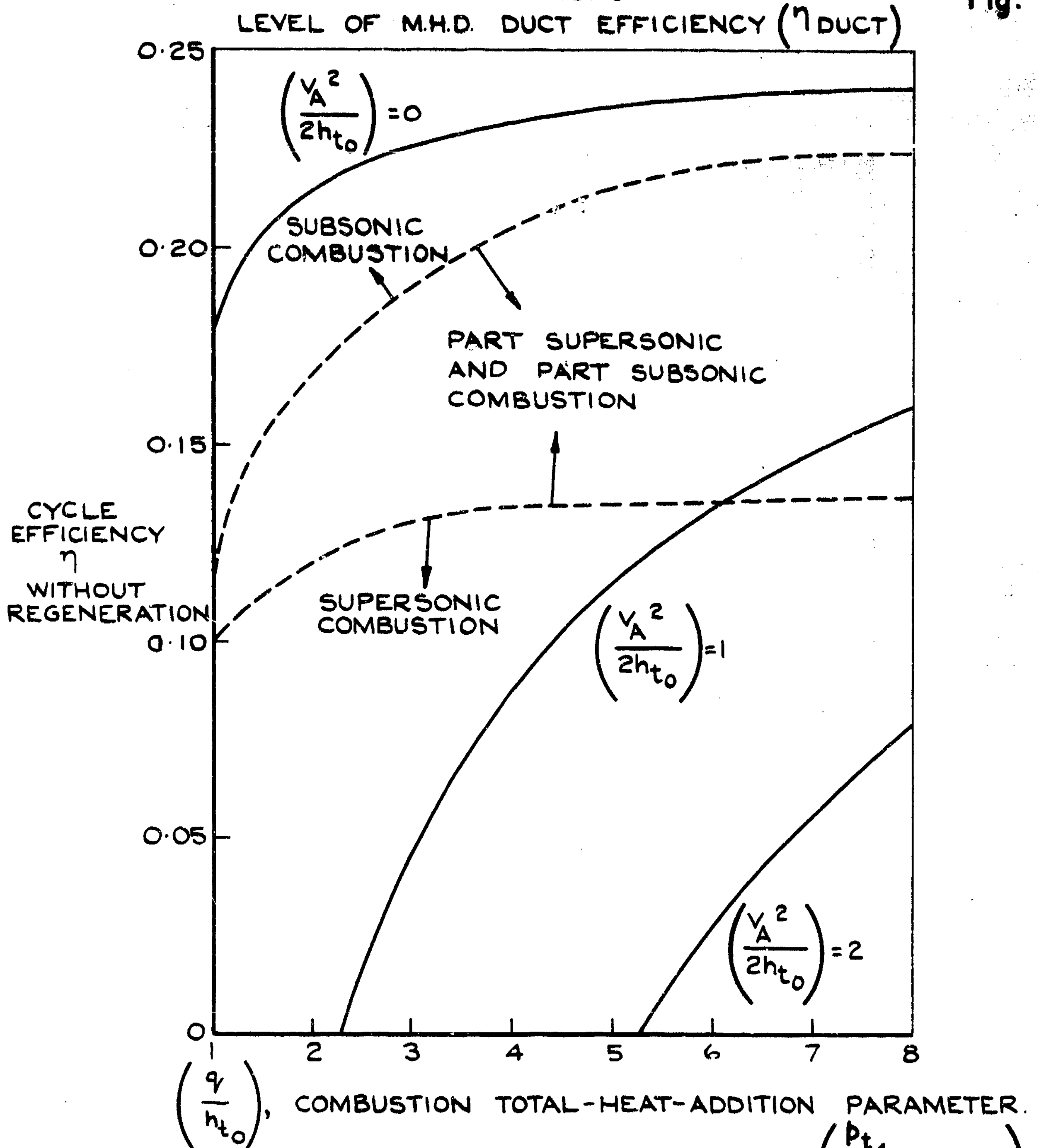

FIG.7a M.H.D DUCT MACH NUMBER 1·4 $\left(\frac{p_{t_4}}{p_{t_5}} = 4{\cdot}66\right)$

FIG.7a-c EFFICIENCY OF A COMPLETE CYCLE WITH CONSTANT PRESSURE (VELOCITY) COMBUSTION AND A CONSTANT MACH NUMBER M.H.D. DUCT (K= 0·75) OF EFFICIENCY 0·25, FOR VARIOUS VALUES OF THE COMBUSTION VELOCITY PARAMETER $\left(\frac{V_A^2}{2h_{t_o}}\right)$ (γ = 1·3)

(SEE APPENDIX E (33) - (37))

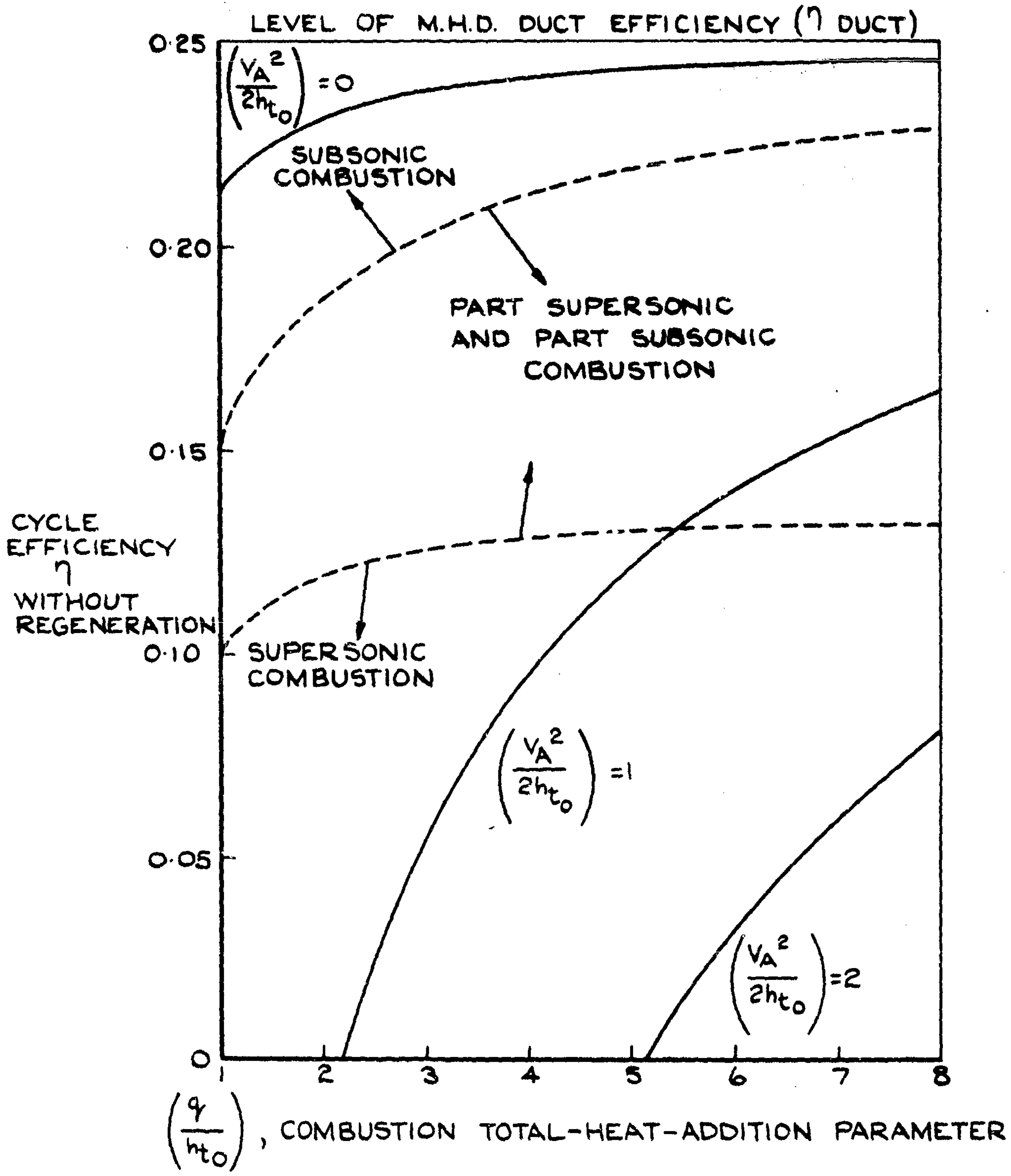

FIG. 7b M.H.D. DUCT MACH NUMBER 0·5 $\left(\frac{p_{t_4}}{p_{t_5}} = 4 \cdot 07\right)$

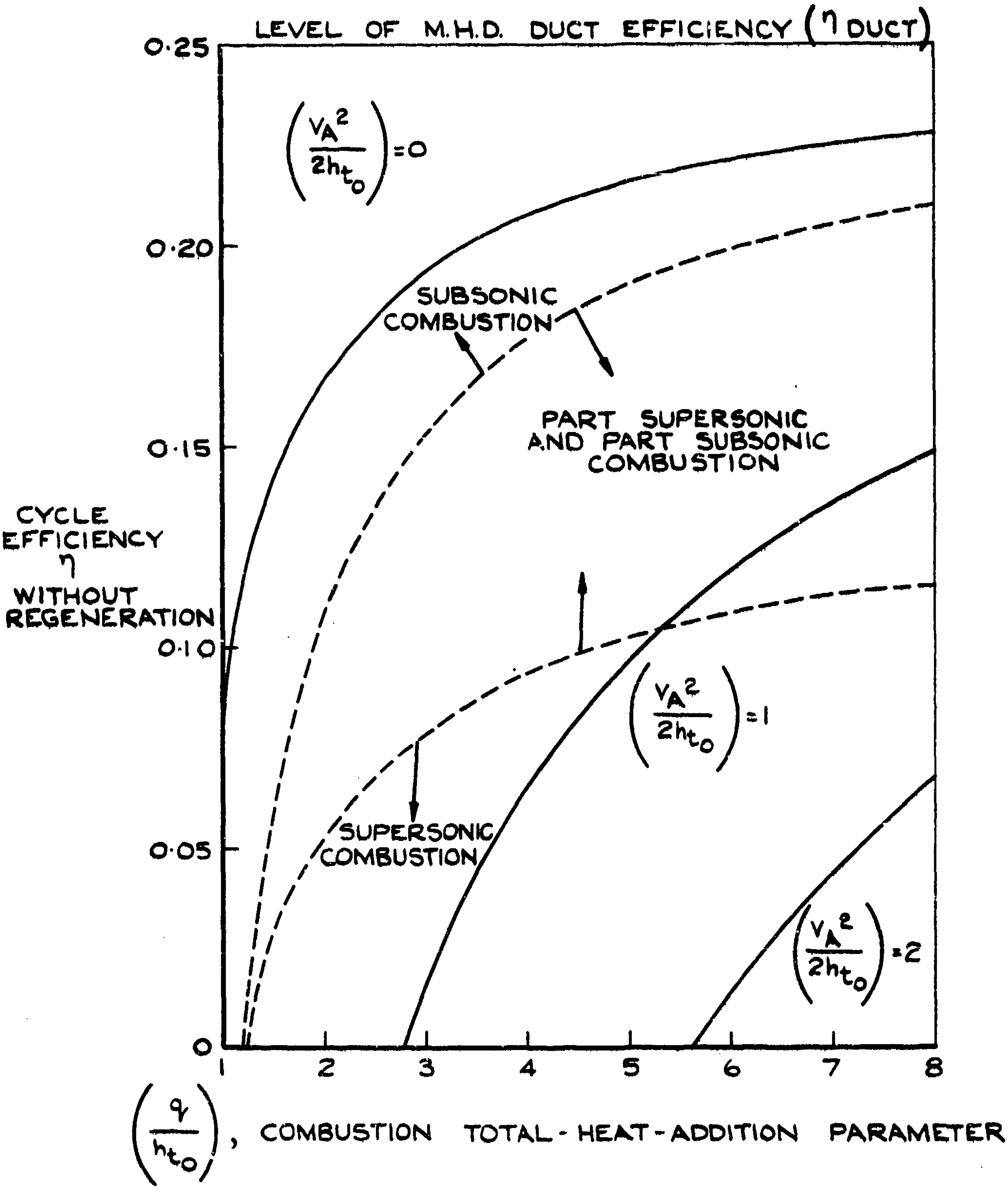

FIG. 7c M.H.D. DUCT MACH NUMBER 2. $\left(\frac{p_{t_4}}{p_{t_5}} = 6·76\right)$

DETACHABLE ABSTRACT CARDS

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In Appendices A to E, for completeness, formulae relevant to the various stages of the cycle are listed or derived.

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